Two-Port Equivalent Circuits of Two-Wire Parabolic Tapered Coupled Transmission Lines

AKIRA ENDO, KUNIKATSU KOBAYASHI, MEMBER, IEEE, YOSHIAKI NEMOTO, MEMBER, IEEE, AND RISABURO SATO, FELLOW, IEEE

Abstract —Two-port equivalent circuits of two-wire parabolic tapered coupled transmission lines (PTCTL) with open or short terminal conditions on the remaining two ports are presented. First, two-port equivalent circuits of PTCTL, whose characteristic admittances increase along the lines, are shown. Second, two-port equivalent circuits of PTCTL, whose characteristic impedances increase along the lines, the dual of the previous circuits, are shown. These two-port circuits of PTCTL are expressed in terms of two equivalent representations, one having mixed lumped and uniform distributed circuits, and the other consisting of uncoupled nonuniform distributed circuits.

I. Introduction

Coupled transmission lines are extremely important in microwave network theory and have been described by many authors [1]–[5]. They are used extensively in all types of microwave components: filters, couplers, matching sections, and equalizers. Nonuniform coupled transmission lines may show good transmission responses and may also be important as microwave components. The analysis of particular nonuniform coupled transmission lines (for instance, exponential or parabolic tapered coupled transmission lines) has been reported [6]–[10]. However, it is very difficult to find exact network functions for general nonuniform coupled transmission lines. One of the most commonly used methods of analyzing coupled transmission lines has been to write the general $n \times n$ immitance matrix, impose appropriate terminal conditions, and reduce the $n \times n$ matrix to its final form, usually a 2×2 matrix. Another method has been to represent equivalent circuits by graph-transformation.

In this paper, by using both of these methods, we obtain two-port equivalent circuits of two-wire parabolic tapered coupled transmission lines (PTCTL). First, two-port equivalent circuits of PTCTL, whose self and mutual characteristic admittances increase along the lines, are shown. These equivalent circuits are expressed as the mixed lumped and distributed circuits consisting of cascade connections of lumped inductors, short-circuited stubs, uncoupled uniform transmission lines, and ideal transformers. These equivalent circuits may also be expressed by the uncoupled nonuniform distributed circuits consisting of parabolic tapered open- and short-circuited stubs and uncoupled parabolic tapered transmission line. Then, two-port equivalent circuits of PTCTL, whose characteristic impedances increase along the lines, the dual of the previous circuit, are also shown. These equivalent circuits can also be expressed as the mixed lumped and distributed circuits, and the uncoupled nonuniform distributed circuits.

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II. Equivalent Representation of Two-Wire Parabolic Tapered Coupled Transmission Lines

Parabolic tapered coupled transmission lines are nonuniform coupled transmission lines whose self and mutual characteristic immitance distributions are given as the parabolic form $(ax+b)^{\pm 2}$, where x is the distance along the line, and a and b are constants. The lossless two-wire PTCTL, whose characteristic admittance distributions are both given by $(ax+b)^2$, is shown in Fig. 1(a), where the line length is l. The equivalent circuit of this PTCTL can be expressed as cascade connections of lumped inductors, uniform coupled transmission lines, and ideal transformers as shown in Fig. 1(b) [7]. We call this circuit an L-type PTCTL. The self and mutual characteristic admittance distributions $Y_{ij}(x)$ of the L-type PTCTL are given by

$$Y_{ij}(x) = y_{ij} \cdot m(x)^2 \qquad (i,j=1,2) \tag{1}$$

$$m(x) = 1 + \left(\frac{W_{11}}{L_{11}} + \frac{W_{12}}{L_{22}}\right)\cdot\frac{x}{l} = 1 + \left(\frac{W_{21}}{L_{11}} + \frac{W_{22}}{L_{22}}\right)\cdot\frac{x}{l} \tag{2}$$

$$m = m(x)|_{x=l} \tag{3}$$

where

- y_{ii} self characteristic admittance of ith transmission line at $x=0$ $(i=1,2)$,
- y_{ij} mutual characteristic admittance between ith and jth transmission lines at $x=0$ $(i,j=1,2)$,
- W_{ii} self characteristic impedance of ith transmission line at $x=0$ $(i=1,2)$,
- W_{ij} mutual characteristic impedance between ith and jth transmission lines at $x=0$ $(i,j=1,2)$,
- L_{ij} inductance of lumped inductor $(i,j=1,2)$,
- m turns ratio of ideal transformer.

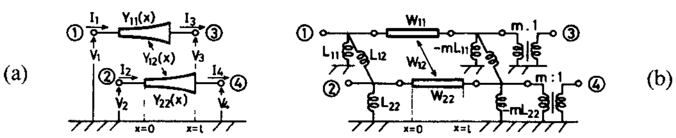


Fig. 1. Two-wire L-type parabolic tapered coupled transmission lines and its equivalent circuit.

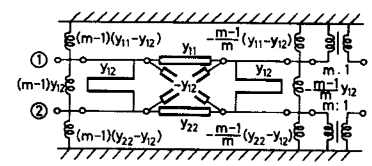


Fig. 2. The equivalent circuit of L-type PTCTL shown in Fig. 1(a).

Here, the element values of the circuit in Fig. 1(b) must satisfy the following condition:

$$\left[\frac{1}{L}\right][W]=(m-1)[I] \tag{4}$$

where $[I]$ is the 2×2 identity matrix, and

$$\left[\frac{1}{L}\right]=\begin{bmatrix} \frac{1}{L_{11}}+\frac{1}{L_{12}} & -\frac{1}{L_{12}} \\ -\frac{1}{L_{12}} & \frac{1}{L_{12}}+\frac{1}{L_{22}} \end{bmatrix} \tag{5}$$

$$[Y]=\begin{bmatrix} y_{11} & -y_{12} \\ -y_{12} & y_{22} \end{bmatrix} \tag{6}$$

$$[W]=[Y]^{-1}=\frac{1}{y_{11}y_{22}-y_{12}^2}\begin{bmatrix} y_{22} & y_{12} \\ y_{12} & y_{11} \end{bmatrix}=\begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix}. \tag{7}$$

Accordingly, $m(x)$ of (2) is obtained as follows:

$$m(x)=1+(m-1)\frac{x}{l}. \tag{8}$$

The two-wire uniform coupled transmission lines can be represented by short-circuited stubs and uncoupled transmission lines [2]. Therefore, the equivalent circuit of the L-type PTCTL in Fig. 1(a) can also be expressed as the mixed lumped and distributed circuits shown in Fig. 2, where the voltages and the currents are related as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix}=\frac{1}{\sqrt{1-p^2}}\begin{bmatrix} A_{11} & 0 & B_{11} & B_{12} \\ 0 & A_{11} & B_{12} & B_{22} \\ C_{11} & C_{12} & D_{11} & 0 \\ C_{12} & C_{22} & 0 & D_{11} \end{bmatrix}\begin{bmatrix} V_3 \\ V_4 \\ I_3 \\ I_4 \end{bmatrix} \tag{9}$$

where

$$A_{11}=m-(m-1)\frac{p}{s} \tag{10}$$

$$B_{ij}=\frac{W_{ij}}{m}p \qquad (i,j=1,2) \tag{11}$$

$$C_{ij}=(-1)^{i+j}\cdot y_{ij}\left\{mp+(m-1)^2\frac{1}{s}-(m-1)^2\frac{p}{s^2}\right\} \qquad (i,j=1,2) \tag{12}$$

$$D_{11}=\frac{1}{m}\left\{1+(m-1)\frac{p}{s}\right\} \tag{13}$$

$$s=j\beta l \tag{14}$$

$$p=j\tan\beta l \quad \text{(Richards variable)} \tag{15}$$

$\beta=$ the phase constant.

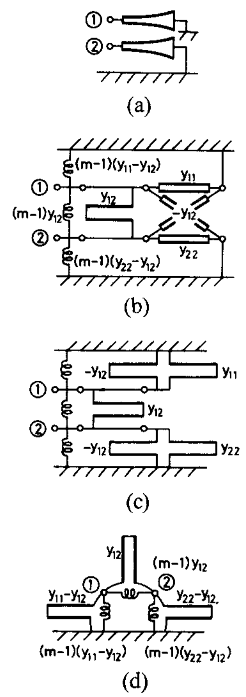


Fig. 3. Equivalent circuit derivation for Example 1 in the text.

III. Two-Port Equivalent Circuits of PTCTL with the Mixed Lumped and Distributed Circuits

Two-port equivalent circuits of PTCTL can be derived by two methods. One is the graph-transformation method. The other is the matrix-reduction method, where a 4×4 chain matrix is reduced to a 2×2 matrix by imposing appropriate terminal conditions.

A. Graph-Transformation Method

To show this method of analysis, two examples are presented next. Example 1 is the network of Fig. 3(a) which has terminals 3 and 4 short-circuited to ground. By using the equivalent representation of Fig. 2 and considering terminal conditions, we obtain the equivalent circuit of Fig. 3(b). The $-y_{12}$ admittances are in parallel with y_{11} and y_{22}, so the graph of Fig. 3(b) reduces to that of Fig. 3(c). Since the unit elements $(y_{11}-y_{12})$ and $(y_{22}-y_{12})$ are short-circuited at one end, they act as short-circuited stubs. The final equivalent circuit of Fig. 3(d) is thus arrived at.

Example 2 is the network of Fig. 4(a), which has terminals 2 and 3 short-circuited to ground. The graph-equivalent circuit is given in Fig. 4(b). By using the same techniques as in the previous example, the equivalent circuit given in Fig. 4(c) and the final circuit in Fig. 4(d) are easily obtained.

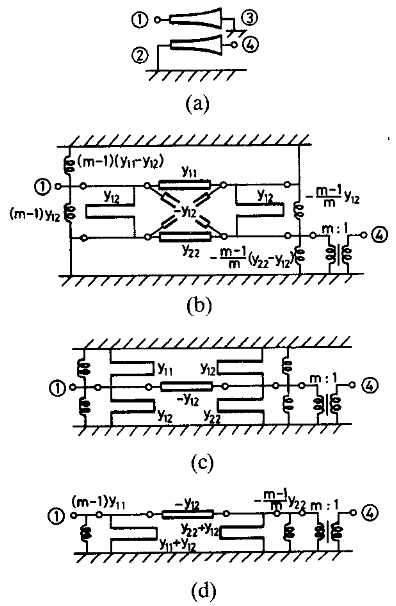


Fig. 4. Equivalent circuit derivation for Example 2 in the text.

B. Matrix-Reduction Method

We apply this method to the network of Fig. 4(a). The terminal conditions in this case are given by

$$\left.\begin{aligned} V_2 &= 0 \\ V_3 &= 0 \end{aligned}\right\}. \tag{16}$$

By substituting (16) in (9), we get

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [F] \begin{bmatrix} V_4 \\ I_4 \end{bmatrix} \tag{17}$$

$$[F] = \frac{1}{\sqrt{1-p^2}} \cdot \frac{1}{B_{12}} \begin{bmatrix} -A_{11}B_{11} & B_{11}B_{22} - B_{12}^2 \\ B_{12}C_{12} - A_{11}D_{11} & -D_{11}B_{22} \end{bmatrix}. \tag{18}$$

There is no formal method for decomposing the chain matrix into realizable submatrices. However, by decomposing (18) appropriately, we finally obtain

$$\begin{aligned}[F] = \frac{1}{\sqrt{1-p^2}} &\begin{bmatrix} 1 & 0 \\ (m-1)y_{11}\frac{1}{s} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (y_{11}+y_{12})\frac{1}{p} & 1 \end{bmatrix} \\ &\cdot \begin{bmatrix} 1 & -\frac{1}{y_{12}}p \\ -y_{12}p & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (y_{12}+y_{22})\frac{1}{p} & 1 \end{bmatrix} \\ &\cdot \begin{bmatrix} 1 & 0 \\ -\frac{m-1}{m}y_{22}\frac{1}{s} & 1 \end{bmatrix} \begin{bmatrix} m & 0 \\ 0 & \frac{1}{m} \end{bmatrix}. \end{aligned} \tag{19}$$

Here, the first and fifth matrices represent parallel lumped inductors, the second and fourth matrices represent parallel short-circuited stubs of length l, the third matrix represents a transmission line of length l, and the sixth matrix represents an ideal transformer. Consequently, the equivalent circuit of Fig. 4(a) can be expressed as the mixed lumped and distributed circuit shown in Fig. 4(d).

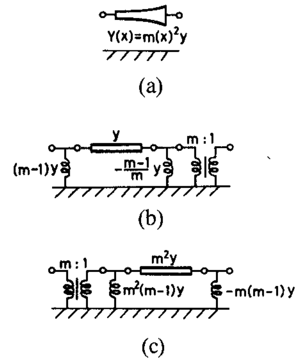


Fig. 5. L-type parabolic tapered transmission line and its equivalent circuit.

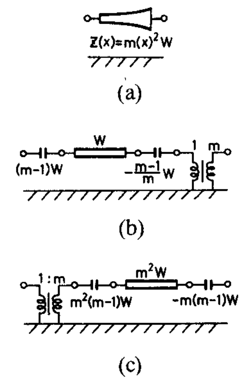


Fig. 6. C-type parabolic tapered transmission line and its equivalent circuit.

In the same manner, two-port equivalent circuits of C-type PTCTL, whose self and mutual characteristic impedance distributions $Z_{ij}(x)$ $(i, j = 1,2)$ increase along the lines, can also be obtained [7]. These equivalent circuits are expressed as the mixed lumped and distributed circuits consisting of cascade connections of lumped capacitors, open-circuited stubs, uncoupled uniform transmission lines, and ideal transformers.

IV. Two-Port Equivalent Circuits of PTCTL with Uncoupled Nonuniform Distributed Circuits

The equivalent circuit of L-type parabolic tapered transmission line (PTTL), shown in Fig. 5(a), may be expressed as cascade connections of lumped inductors, a uniform transmission line, and an ideal transformer shown in Fig. 5(b) and (c) [11]. Similarly, the equivalent circuit of C-type PTTL, shown in Fig. 6(a), may be expressed as cascade connections of lumped capacitors, a uniform transmission line, and an ideal transformer shown in Fig. 6(b) and (c) [11]. So, two-port equivalent circuits of PTCTL may be represented with uncoupled parabolic tapered transmission lines only.

The L-type parabolic tapered open- or short-circuited stubs and their equivalent circuits are shown in Table I, and C-type parabolic tapered stubs and their equivalent circuits are shown in Table II, respectively. Representa-

TABLE I
L-TYPE PARABOLIC TAPERED STUBS AND THEIR EQUIVALENT CIRCUITS

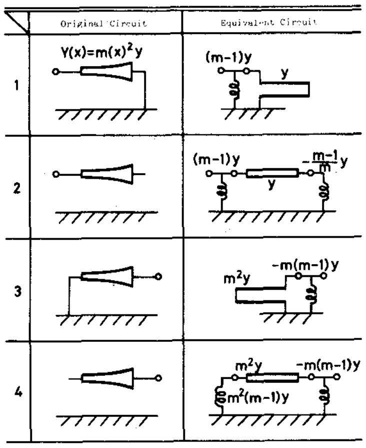


TABLE II
C-TYPE PARABOLIC TAPERED STUBS AND THEIR EQUIVALENT CIRCUITS

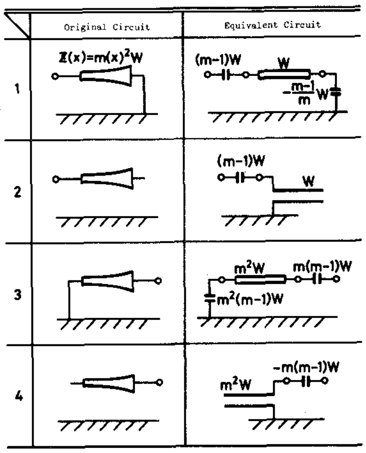


TABLE III
NOTIONS OF SERIES AND PARALLEL PARABOLIC TAPERED STUBS

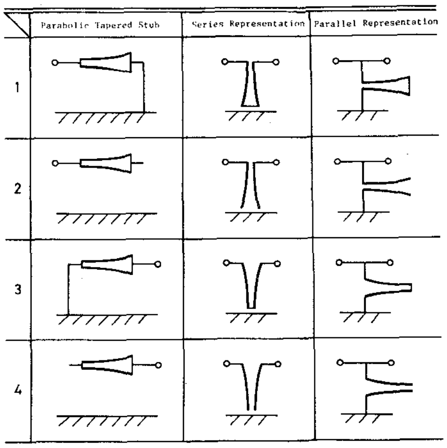


TABLE IV
TWO-PORT EQUIVALENT CIRCUITS OF *L*-TYPE PTCTL

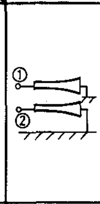
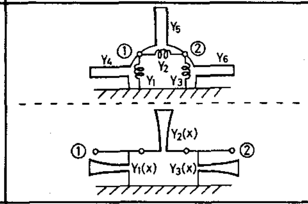
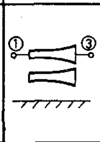
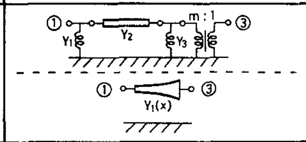
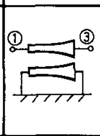
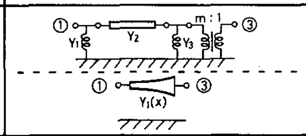
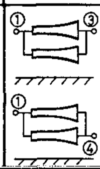
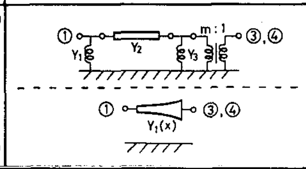
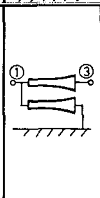
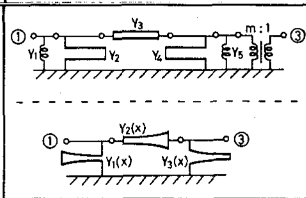
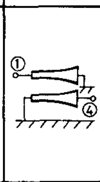
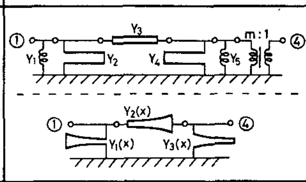
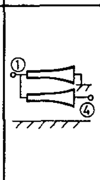
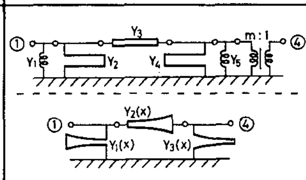
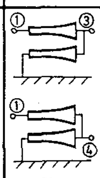
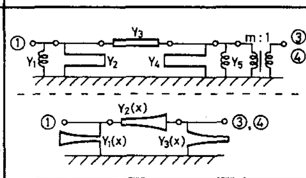
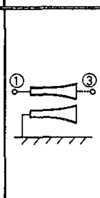
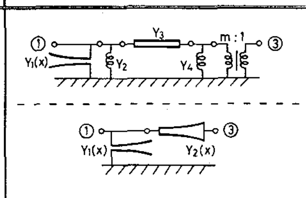
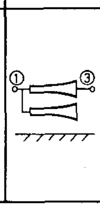
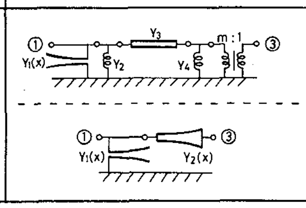
	Original Circuit	Equivalent Circuits	Element Values
1			$Y_1=(m-1)(y_{11}-y_{12})$, $Y_4=y_{11}-y_{12}$ $Y_2=(m-1)y_{12}$, $Y_5=y_{12}$ $Y_3=(m-1)(y_{22}-y_{12})$, $Y_6=y_{22}-y_{12}$ $Y_1(x)=m(x)^2(y_{11}-y_{12})$ $Y_2(x)=m(x)^2y_{12}$ $Y_3(x)=m(x)^2(y_{22}-y_{12})$
2			$Y_1=(m-1)\frac{\Delta y}{y_{22}}$, $Y_2=\frac{\Delta y}{y_{22}}$ $Y_3=-\frac{m-1}{m}\frac{\Delta y}{y_{22}}$ $Y_1(x)=m(x)^2\frac{\Delta y}{y_{22}}$
3			$Y_1=(m-1)y_{11}$, $Y_2=y_{11}$ $Y_3=-\frac{m-1}{m}y_{11}$ $Y_1(x)=m(x)^2y_{11}$
4			$Y_1=(m-1)(y_{11}+y_{22}-2y_{12})$ $Y_2=y_{11}+y_{22}-2y_{12}$ $Y_3=-\frac{m-1}{m}(y_{11}+y_{22}-2y_{12})$ $Y_1(x)=m(x)^2(y_{11}+y_{22}-2y_{12})$
5			$Y_1=(m-1)(y_{11}+y_{22}-2y_{12})$, $Y_4=y_{12}$ $Y_2=y_{22}-y_{12}$ $Y_3=y_{11}-y_{12}$, $Y_5=-\frac{m-1}{m}y_{11}$ $Y_1(x)=m(x)^2(y_{22}-y_{12})$ $Y_2(x)=m(x)^2(y_{11}-y_{12})$ $Y_3(x)=m(x)^2y_{12}$
6			$Y_1=(m-1)y_{11}$, $Y_4=y_{22}+y_{12}$ $Y_2=y_{11}+y_{12}$ $Y_3=-y_{12}$, $Y_5=-\frac{m-1}{m}y_{22}$ $Y_1(x)=m(x)^2(y_{11}+y_{12})$ $Y_2(x)=-m(x)^2y_{12}$ $Y_3(x)=m(x)^2(y_{22}+y_{12})$
7			$Y_1=(m-1)(y_{11}+y_{22}-2y_{12})$, $Y_4=y_{12}$ $Y_2=y_{11}-y_{12}$ $Y_3=y_{22}-y_{12}$, $Y_5=-\frac{m-1}{m}y_{22}$ $Y_1(x)=m(x)^2(y_{11}-y_{12})$ $Y_2(x)=m(x)^2(y_{22}-y_{12})$ $Y_3(x)=m(x)^2y_{12}$
8			$Y_1=(m-1)y_{11}$, $Y_4=y_{22}-y_{12}$ $Y_2=y_{12}$, $Y_5=-\frac{m-1}{m}(y_{11}+y_{22}-2y_{12})$ $Y_3=y_{11}-y_{12}$ $Y_1(x)=m(x)^2y_{12}$ $Y_2(x)=m(x)^2(y_{11}-y_{12})$ $Y_3(x)=m(x)^2(y_{22}-y_{12})$
9			$Y_1(x)=m(x)^2\frac{y_{12}^2}{y_{22}}$, $Y_3=\frac{\Delta y}{y_{22}}$ $Y_2=(m-1)\frac{\Delta y}{y_{22}}$, $Y_4=-\frac{m-1}{m}\frac{\Delta y}{y_{22}}$ $Y_1(x)=m(x)^2\frac{y_{12}^2}{y_{22}}$ $Y_2(x)=m(x)^2\frac{\Delta y}{y_{22}}$
10			$Y_1(x)=m(x)^2\frac{(y_{22}-y_{12})^2}{y_{22}}$, $Y_3=\frac{\Delta y}{y_{22}}$ $Y_2=(m-1)\frac{\Delta y}{y_{22}}$, $Y_4=-\frac{m-1}{m}\frac{\Delta y}{y_{22}}$ $Y_1(x)=m(x)^2\frac{(y_{22}-y_{12})^2}{y_{22}}$ $Y_2(x)=m(x)^2\frac{\Delta y}{y_{22}}$

TABLE IV (*Continued*)

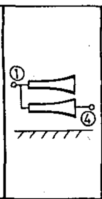
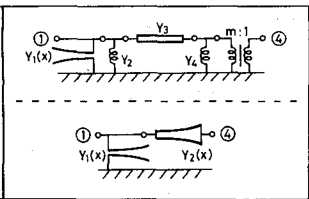
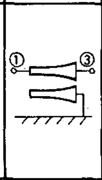
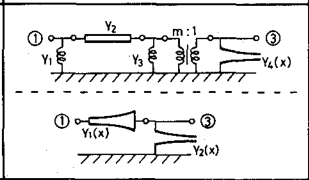
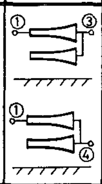
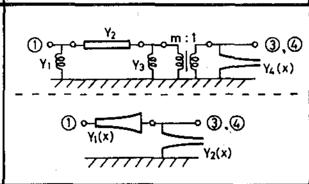
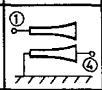
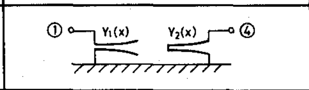
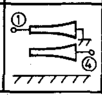
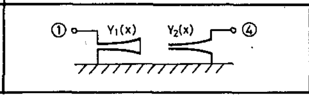
11			$Y_1(x)=m(x)^2 \frac{(y_{11}-y_{12})^2}{y_{11}}$, $Y_3=\frac{\Delta y}{y_{11}}$ $Y_2=(m-1)\frac{\Delta y}{y_{11}}$, $Y_4=-\frac{m-1}{m}\frac{\Delta y}{y_{11}}$ $Y_1(x)=m(x)^2 \frac{(y_{11}-y_{12})^2}{y_{11}}$ $Y_2(x)=m(x)^2 \frac{\Delta y}{y_{11}}$
12			$Y_1=(m-1)\frac{\Delta y}{y_{22}}$, $Y_3=-\frac{m-1}{m}\frac{\Delta y}{y_{22}}$ $Y_2=\frac{\Delta y}{y_{22}}$, $Y_4(x)=m(x)^2\frac{y_{12}^2}{y_{22}}$ $Y_1(x)=m(x)^2\frac{\Delta y}{y_{22}}$ $Y_2(x)=m(x)^2\frac{y_{12}^2}{y_{22}}$
13			$Y_1=(m-1)\frac{\Delta y}{y_{22}}$, $Y_3=-\frac{m-1}{m}\frac{\Delta y}{y_{22}}$ $Y_2=\frac{\Delta y}{y_{22}}$, $Y_4(x)=m(x)^2\frac{(y_{22}-y_{12})^2}{y_{22}}$ $Y_1(x)=m(x)^2\frac{\Delta y}{y_{22}}$ $Y_2(x)=m(x)^2\frac{(y_{22}-y_{12})^2}{y_{22}}$
14			$Y_1(x)=m(x)^2 y_{11}$ $Y_2(x)=m(x)^2\frac{\Delta y}{y_{11}}$
15			$Y_1(x)=m(x)^2\frac{\Delta y}{y_{22}}$ $Y_2(x)=m(x)^2 y_{22}$

TABLE V
TWO-PORT EQUIVALENT CIRCUITS OF *C*-TYPE PTCTL

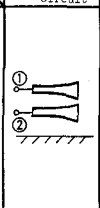
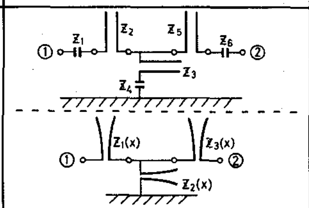
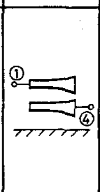
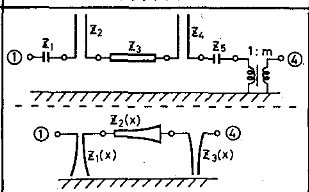
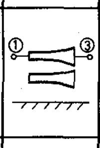
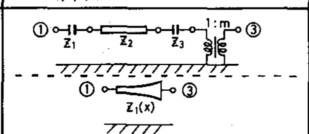
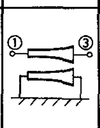
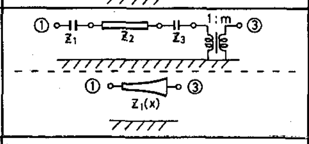
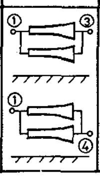
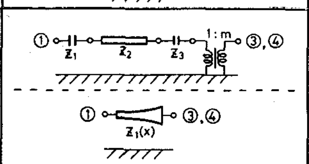
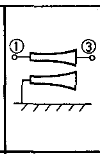
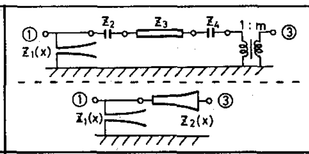
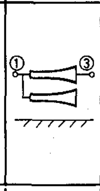
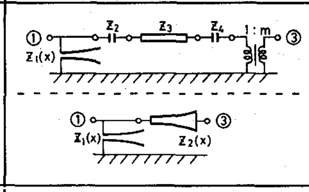
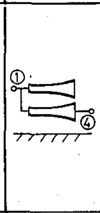
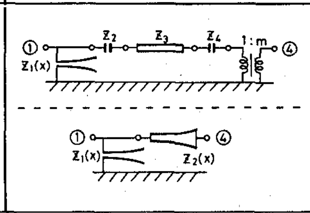
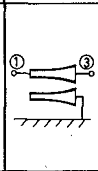
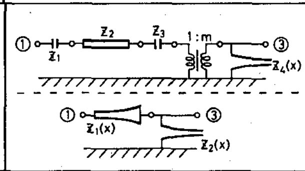
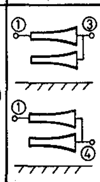
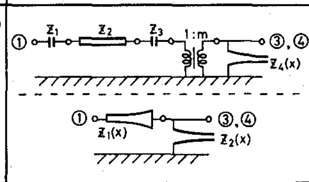
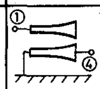
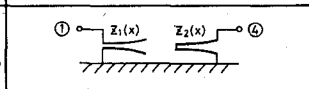
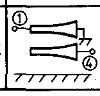
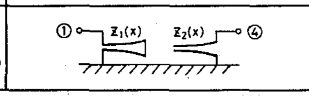
	Original Circuit	Equivalent Circuits	Element Values
1			$Z_1=(m-1)(W_{11}-W_{12})$, $Z_2=W_{11}-W_{12}$ $Z_3=(m-1)W_{12}$, $Z_4=W_{12}$ $Z_5=(m-1)(W_{22}-W_{12})$, $Z_6=W_{22}-W_{12}$ $Z_1(x)=m(x)^2(W_{11}-W_{12})$ $Z_2(x)=m(x)^2W_{12}$ $Z_3(x)=m(x)^2(W_{22}-W_{12})$
2			$Z_1=(m-1)W_{11}$, $Z_4=W_{22}-W_{12}$ $Z_2=W_{11}-W_{12}$, $Z_5=-\frac{m-1}{m}W_{22}$ $Z_3=W_{12}$ $Z_1(x)=m(X)^2(W_{11}-W_{12})$ $Z_2(x)=m(x)^2W_{12}$ $Z_3(x)=m(x)^2(W_{22}-W_{12})$
3			$Z_1=(m-1)W_{11}$, $Z_2=W_{11}$ $Z_3=-\frac{m-1}{m}W_{11}$ $Z_1(x)=m(x)^2W_{11}$
4			$Z_1=(m-1)\frac{\Delta W}{W_{22}}$, $Z_2=\frac{\Delta W}{W_{22}}$ $Z_3=-\frac{m-1}{m}\frac{\Delta W}{W_{22}}$ $Z_1(x)=m(x)^2\frac{\Delta W}{W_{22}}$
5			$Z_1=(m-1)\frac{\Delta W}{W_{11}+W_{22}-2W_{12}}$ $Z_2=\frac{\Delta W}{W_{11}+W_{22}-2W_{12}}$ $Z_3=-\frac{m-1}{m}\frac{\Delta W}{W_{11}+W_{22}-2W_{12}}$ $Z_1(x)=m(x)^2\frac{\Delta W}{W_{11}+W_{22}-2W_{12}}$

TABLE V (*Continued*)

6			$Z_1(x)=m(x)^2\frac{W_{11}}{W_{12}^2}\Delta W$, $Z_3=W_{11}$ $Z_2=(m-1)W_{11}$, $Z_4=-\frac{m-1}{m}W_{11}$ $Z_1(x)=m(x)^2\frac{W_{11}}{W_{12}^2}\Delta W$ $Z_2(x)=m(x)^2W_{11}$
7			$Z_1(x)=m(x)^2\frac{W_{11}}{(W_{11}-W_{12})^2}\Delta W$ $Z_2=(m-1)W_{11}$ $Z_4=-\frac{m-1}{m}W_{11}$, $Z_3=W_{11}$ $Z_1(x)=m(x)^2\frac{W_{11}}{(W_{11}-W_{12})^2}\Delta W$ $Z_2(x)=m(x)^2W_{11}$
8			$Z_1(x)=m(x)^2\frac{W_{22}}{(W_{22}-W_{12})^2}\Delta W$ $Z_2=(m-1)W_{22}$, $Z_3=W_{22}$ $Z_4=-\frac{m-1}{m}W_{22}$ $Z_1(x)=m(x)^2\frac{W_{22}}{(W_{22}-W_{12})^2}\Delta W$ $Z_2(x)=m(x)^2W_{22}$
9			$Z_1=(m-1)W_{11}$, $Z_3=-\frac{m-1}{m}W_{11}$ $Z_2=W_{11}$. $Z_4(x)=m(x)^2\frac{W_{11}}{W_{12}^2}\Delta W$ $Z_1(x)=m(x)^2W_{11}$ $Z_2(x)=m(x)^2\frac{W_{11}}{W_{12}^2}\Delta W$
10			$Z_1=(m-1)W_{11}$, $Z_3=-\frac{m-1}{m}W_{11}$ $Z_2=W_{11}$, $Z_4(x)=m(x)^2\frac{\Delta W \cdot W_{11}}{(W_{11}-W_{12})^2}$ $Z_1(x)=m(x)^2W_{11}$ $Z_2(x)=m(x)^2\frac{W_{11}}{(W_{11}-W_{12})^2}\Delta W$
11			$Z_1(x)=m(x)^2\frac{\Delta W}{W_{22}}$ $Z_2(x)=m(x)^2W_{22}$
12			$Z_1(x)=m(x)^2W_{11}$ $Z_2(x)=m(x)^2\frac{\Delta W}{W_{11}}$

tions of series parabolic tapered stubs and parallel parabolic tapered stubs are shown in Table III. By using parabolic tapered stubs and PTTL, two-port equivalent circuits of PTCTL may also be expressed by the uncoupled nonuniform distributed circuits.

V. EXAMPLES OF TWO-PORT EQUIVALENT CIRCUITS OF PTCTL

In Table IV, two-port equivalent circuits of the *L*-type PTCTL, one having mixed lumped and uniform distributed circuits and the other consisting of uncoupled parabolic tapered transmission lines, are introduced. Similarly, two-port equivalent circuits of the *C*-type PTCTL are shown in Table V.

VI. CONCLUSION

Fifteen equivalent circuits of *L*-type PTCTL and twelve equivalent circuits of *C*-type PTCTL have been derived using the graph-transformation method and the matrix-reduction method. Two-port equivalent circuits of *L*-type PTCTL are expressed as the mixed lumped and distributed circuits consisting of cascade connections of lumped inductors, short-circuited stubs, uncoupled uniform transmission lines, and ideal transformers, and may also be expressed by the uncoupled nonuniform distributed circuits consisting of parabolic tapered open- and short-circuited stubs and uncoupled PTTL. Similarly, two-port circuits of C-type PTCTL, the dual of the previous circuits, can also be expressed by two equivalent representations of the mixed lumped and distributed circuits, and the uncoupled nonuniform distributed circuits.

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Akira Endo was born in Yamagata, Japan, on February 14, 1958. He received the B.E. and M.E. degrees from Yamagata University, Yonezawa, Japan, in 1980 and 1983, respectively.

He has been engaged in research works in nonuniform coupled transmission lines.

Mr. Endo is a member of the Institute of Electronics and Communication Engineers of Japan.

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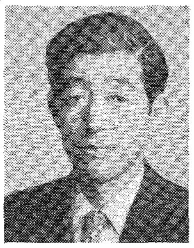
From 1971 to 1975, he was a Research Associate with the Faculty of Engineering, Yamagata University, and in 1975 he became a Lecturer at the same university. He has been engaged in research works in mixed lumped and distributed networks. He was co-recipient of the 1982 Microwave Prize from the IEEE MTT-S.

Mr. Kobayashi is a member of the Institute of Electronics and Communication Engineers of Japan.

Yoshiaki Nemoto (S'72–M'73) was born in Sendai City, Miyagiken, Japan, on December 2, 1945. He received the B.E., M.E., and Ph.D. degrees from Tohoku University, Sendai, Japan, in 1968, 1970, and 1973, respectively.

Since 1973, he has been a Research Associate with the Faculty of Engineering, Tohoku University. He has been engaged in research works in distributed networks and computer networks using satellites. He is co-recipient of the 1982 Microwave Prize from the IEEE Microwave Theory and Techniques Society.

Dr. Nemoto is a member of the Institute of Electronics and Communication Engineers of Japan.



Risaburo Sato (SM'62–F'77) was born in Furukawa City, Miyagiken, Japan, on September 23, 1921. He received the B.E. and the Ph.D. degrees from Tohoku University, Sendai, Japan, in 1944 and 1952, respectively.

From 1949 to 1961, he was an Assistant Professor at Tohoku University, and in 1961 he became a Professor in the Department of Electrical Communications at the same university. Since 1973, he has been a Professor in the Department of Information Science at Tohoku University. From 1969 to 1970, he was an International Research Fellow at Stanford Research Institute, Menlo Park, CA. His research activities include studies of multiconductor transmission systems, distributed transmission circuits, antennas, communication systems, active transmission lines, magnetic and ferroelectric recording, neural information processing, computer networks, and electromagnetic compatibility. He has published a number of technical papers and some books in these fields, including *Transmission Circuit*. He received the Paper Award from the Institute of Electrical Engineers of Japan (IEE of Japan) in 1955, the Kahoku Press Cultural Award in 1963, an award from the Invention Association of Japan in 1966, the Paper Award from the Institute of Electronics and Communication Engineers of Japan (IECE of Japan) in 1980, a Certificate of Appreciation of Electromagnetic Compatibility from the IEEE in 1981, and the Microwave Prize of the Microwave Theory and Techniques Society of IEEE in 1982.

Dr. Sato was the Vice President of IECE of Japan from 1974 to 1976. He has been a member of the Science Council of Japan from 1978 and a member of the Telecommunication Technology Consultative Committee at NTT from 1976. He is a chairman of EMC-S Tokyo Chaper of IEEE and a member of B.O.D. of EMC-S of IEEE. He is also a member of IECE of Japan, IEE of Japan, the Institute of Television Engineers of Japan, and the Information Processing Society of Japan.